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Almost Contra αGS-Continuous Functions

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Abstract— The objective of this paper is to introduce and study a new generalization of contra continuity called almost contra αgs-continuous functions using αgs-open sets. We discuss the relationships with some other related functions.

Index Terms— αgs-closed set, almost contra αgs-continuous, contra αgs-continuous, $T_{\alpha gs}$-space , αgs-converging, αgs-connected, αGS-regular graphs

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1 Introduction

General Topology plays an important role in many fields of applied sciences as well as branches of mathematics. In reality it is used in computational topology for geometric design and molecular design, computer-aided design, comuter-aided geometric design, digital topology, data mining, information systems and qumtum physics. One can refer to the following papers, respectively: [16], [25], [12-13],[21],[14], [8-11].

In 1996, Dontchev [5] introduced the notion of contra continuity and strong S-closedness in topological spaces. A new weaker form of this class of functions called contra semi continuous function is introduced and investigated by Dontchev and Noiri [6]. Recently, Rajamani and Vishwanathan [22] have introduced the notion of αgs-closed sets. Using αgs-closed set we define and study new class of functions called Almost contra αgs-continuous as well as contra αgs-continuous functions as a new generalization of contra continuity**.**

2 Prilimnaries

Throughout this paper (X,τ) and (Y,σ) (or simply X and Y) denote topological spaces on which no separation axioms are assumed unless explicitly stated. If A is any subset of space X, then Cl(A) and Int(A) denote the closure of A and the interior of A in X respectively.
The following definitions are useful in the sequel:

Definition 2.1: A subset A of space X is called
(i) a semi-open set [15] if $A \subseteq Cl(Int(A))$
(ii) a semi-closed set [1] if $Int(Cl(Int(A))) \subseteq A$
(iii) α-open [18] if $A \subseteq Int(Cl(Int(A)))$

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(iii) a regular open set [30] if A = Int(Cl(A))

Definition 2.2 [22]**:** A subset A of a topological space X is called α-generalized-semi closed (briefly, αgs-closed) if $\alpha cl(A) \subset U$, whenever $A \subset U$ and U is semi-open in X. The complement of αgs-closed set is α-generalized-semi open (briefly, αgs-open). We denote the family of αgs-closed sets of X by αGSC(X, τ) and αgs-open sets by αGSO(X, τ).

Definition 2.3 [17]: A topological space X is said to be
(i) α-generalized semi-T_o (in brief, αgs-T_o), if for each pair of distinct points x, y of X, there exists an αgs-open set containing one point but not the other.
(ii) α-generalized semi-T_1 (in brief, αgs-T_1), if for each pair of distinct points x, y of X, there exist disjoint αgs-open sets, one containing x but not y and the other containing y but not x.
(iii) α-generalized semi-T_2 (in brief, αgs-T_2), if for each pair of distinct points x, y of X, there exist disjoint αgs-open sets U and V such that $x \in U$ and $y \in V$.

Definition 2.4 [23]: A function $f: X \to Y$ is called α-generalized semi-continuous (in briefly, αgs-continuous), if $f^{-1}(F)$ is αgs-closed in X for every closed set F of Y.

Definition 2.5 [23]: A space X is called $T_{\alpha gs}$-space if every αgs-closed set in in is closed set.

Definition 2.6 [26]
(i) Nearly countably compact if every countable cover of X by regular open sets has a finite subcover.
(ii) Nearly compact if every regular open cover of X has a finite subcover.
(iii) S-closed [31] if every regular closed cover of x has a finite Subcover

3 Almost Contra αGS-Continuous Functions

In this section we introduce new type of continuity called almost contra αgs-continuous functions which is weaker than contra αgs-continuous.

Definition 3.1: A function $f: X \to Y$ is said to be almost contra αgs-continuous if $f^{-1}(V)$ is αgs-closed set in X for each regular open set V of Y.

Definition 3.2: A function $f: X \to Y$ is called contra αgs-continous function if $f^{-1}(F)$ is αgs-closed in X for every open set

F of Y.

Theorem 3.3: If f: X→Y is contra αgs-continuous then it is almost contra αgs-continuous.
Proof: Obvious, as every regular open set is open set.

But converse of the above theorem is not true as shown by the following example.

Example 3.4: Let X =Y = {a, b, c}, τ = {X, φ, {a}, {b, c}}, σ = {Y, φ, {a}, {c}, {a, c}}. We have αgs -closed sets in X are {{b}, {c}, {a, b}, {a, c}}. Then f: X → Y defined by f(a) = b, f(b) = a, f(c) = c is almost contra αgs-continuous but not contra αgs-continuous function as for open set {a, c} in Y, f^{-1}({a, c}) = {b ,c} which is not αgs-closed set in X.

Definition 3.5: A space X is called locally αgs-indiscrete if every αgs-open set is closed in X.

Theorem 3.6: If f: X→Y is almost contra αgs-continuous and X is locally αgs-indiscrete space then f is almost contra continuous.

Proof: Let V be a regular open set in Y. Since f is almost contra αgs-continuous f^{-1}(V) is αgs-closed set in X and X is locally αgs-indiscrete space, which implies f^{-1}(V) is an open set in X. Therefore f is almost contra continuous.

Theorem 3.7: If f: X→Y is almost contra αgs-continuous and X is $T_{\alpha gs}$-space then f is almost continuous.
Proof: Let V be a regular open set in Y. Since f is almost contra αgs-continuous f^{-1}(V) is αgs-closed set in X and X is $T_{\alpha gs}$-space, which implies f^{-1}(V) is closed set in X. Therefore f is almost contra continuous.

Although αGSC(X) is closed under finite union, αGSO(X) is not. In the following theorem, we assume that αGSC(X) is closed under arbitrary union.

Theorem 3.8: The following are equivalent for a function f: X→Y
(i) f is almost contra αgs-continuous.
(ii) For every regular closed set F of Y, f^{-1}(F) is αgs-open set of X.
(iii) for each x ∈ X and each regular closed set F of Y containing f(x), there exists αgs-open set U containing x such that f(U) ⊂ F.
(iv) for each x ∈ X and each regular closed set V of Y not containing f(x), there exists αgs-closed set K not containing x such that f^{-1}(V) ⊂ K..

Proof: (i) ⇒(ii) Let F be a regular closed set of Y. Then Y-F is regular open set in Y. By (i), f^{-1}(Y-F) = X-f^{-1}(F) is αgs -closed set in X. This implies f^{-1}(F) is αgs -open set in X. Therefore, (ii) holds.

(ii) ⇒(i) Let G be an regular open set of Y. Then Y-G is a regular closed set in Y. By (ii), f^{-1}(Y-G) is αgs-open set in X. This implies X-f^{-1}(G) is αgs-open set in X. this implies f^{-1}(G) is αgs -closed set in X. Therefore, (i) hold.

(ii) ⇒(iii) Let F be a regular closed set of Y containing f(x), which implies x ∈ f^{-1}(F). By (ii) f^{-1}(F) is αgs-open set in X containing x. Set U = f^{-1}(F), which implies U is αgs-open set in X containing x and f(U) = f(f^{-1}(F)) ⊂ F . Therefore, (iii) holds.

(iii) ⇒(ii) Let F be a regular closed set in Y containing f(x), which implies x ∈ f^{-1}(F). By (iii) there exists αgs-open set U_x in X containing x such that. f(U) ⊂ F that is U ⊂ f^{-1}(F) =∪ {U_x : x ∈ f^{-1}(F)} which is union of αgs-open sets. Therefore f^{-1}(F) is αgs-open set of X.

(iii) ⇒(iv) Let F be a regular open set of Y not containing f(x). Then Y-V is a regular closed set in Y containing f(x). From (iii), there exists a αgs-open set U in X containing x such that f(U) ⊂ Y-V. This implies that U ⊂ f^{-1}(Y-V) = X - f^{-1}(V). Hence, f^{-1}(V) ⊂ X-U. Set X-U =K, then K is αgs-closed set not containing x in X such that f^{-1}(V) ⊂ K.

(iv) ⇒(iii) Let F be a regular closed set in Y containing f(x). Then Y-f(x) is a regular open set in Y not containing f(x). From (iv), there exists a αgs-closed set K in X not containing x such that f^{-1}(Y-F) ⊂ K. This implies that X- f^{-1}(F) ⊂ K. Hence X-K ⊂ f^{-1}(F), that is f(X-K) ⊂ F. Set U = X- K, then U is αgs-open set containing x in X such that f(U) ⊂ F.

Recall that space X is said to be weakly Hausdorff [27] if each element of X is an intersection of regular closed sets.

Theorem 3.9: If f: X→Y is an almost contra αgs-continuous injection and Y is weakly Hausdorff, then X is αgs-T_1.

Proof: Suppose Y is weakly Hausdorff. For any distinct points x and y in X, there exist V and W regular closed sets in Y such that f(x)∈V, f(y)∉V, f(y)∈W and f(x)∉W. Since f is almost contra αgs-continuous, implies f^{-1}(V) and f^{-1}(W) are αgs-open subsets of X such that x∈f^{-1}(V), y∉f^{-1}(V), y∈f^{-1}(W) and x∉ f^{-1}(W). This shows that X is αgs-T_1.

Corollary 3.10: If f: X→Y is a contra αgs-continuous injection and Y is weakly Hausdorff, then X is αgs-T_1.

Definition 3.11[28]: A topological space X is called Ultra Hausdroff space, if for every pair of distinct points x and y in X, there exist disjoint clopen sets U and V in X containing x and y respectively.

Theorem 3.12: If f: X→Y is an almost contra αgs-continuous injective function from space X into an Ultra Hausdroff space Y, then X is αgs-T_2.

Proof: Let x and y be any two distinct points in X. Since f is

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injective f(x) ≠ f(y) and Y is Ultra Hausdroff space, implies there exist disjoint clopen sets U and V of Y containing f(x) and f(y) respectively. Then $x \in f^{-1}(U)$ and $y \in f^{-1}(V)$, where $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint αgs-open sets in X. Therefore X is αgs-T_2.

Definition 3.13: A topological space X is said to be
(i) ultra normal [28]if each pair of disjoint closed sets can be separated by disjoint clopen sets.
(ii) αgs-normal [24] if for any pair of disjoint αgs-closed sets A and B in X, there exist disjoint open sets U and V in X such that $A \subseteq U$ and $B \subseteq V$.

Theorem 3.14: If f: X→Y is an almost contra αgs-continuous closed injection and Y is ultra normal, then X is αgs-normal.

Proof: Let E and F be disjoint closed subsets of X. Since f is closed and injective f(E) and f(F) are disjoint closed sets in Y. Since Y is ultra normal there exists disjoint clopen sets U and V in Y such that $U \subset f(E)$ and $V \subset f(F)$. This implies $f^{-1}(U) \subset E$ and $f^{-1}(V) \subset F$. Since f is almost contra αgs-continuous injection, $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint αgs-open sets in X. This shows X is αgs-normal.

Definition 3.15: A filter base **Λ** is said to be αgs-convergent (resp. rc-covergent [7]) to a point x ∈ X if for any A ∈ αGSO(X) containing x (resp. A ∈ RC(X) containing x), there exists a B∈ **Λ** such that B ⊂ A.

Theorem 3.16: : If f: X→Y is an almost contra αgs-continuous, then for each point x ∈ X and each filter base **Λ** in X αgs-converging to x, the filter base f(**Λ**) is rc-convrgent to f(x).

Proof: Let x ∈ X and **Λ** be any filterbase in X αgs-converging to x. Since f is almost contra αgs-continuous, then for any V∈ RC(Y) containing f(x), there exists U ∈ αGSO(X) containing x such that f(U) ⊂ V. Since **Λ** is αgs-convergent to x, there exists a B ∈ **Λ** such that B ⊂ U. This means that f(B) ⊂ V and therefore the filterbase f(**Λ**) is rc-convergent to f(x).

Theorem 3.17: If f: X→Y is an almost contra αgs-continuous closed injection and A is open subset of X, then the restriction (f/A): X→Y is almost contra αgs-continuous.

Proof: Let F be a regular closed set in Y. Since f is almost contra αgs-continuous, $f^{-1}(F) \in$ αGSO(X). Since A is open, it follows that $(f/A)^{-1}(F) = A \cap f^{-1}(F) \in$ αGSO(X). Therefore (f/A): X→Y is almost contra αgs-continuous.

Now, let us define the following.

Definition 3.18: A topological space X is said to be αgs-connected if X cannot be expressed as disjoint union two non empty αgs-open sets

Theorem 3.19: If f: X→Y is an almost contra αgs continuous surjection and X is αgs-connected, then Y is connected.

Proof: Suppose Y is a not connected space. Then there exist disjoint open sets U and V such that $Y=U \cup V$. Therefore U and V are clopen in Y. Since f is an almost contra αgs-continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are αgs-open sets in X. Moreover $f^{-1}(U)$ and $f^{-1}(V)$ are non empty disjoint and $X= f^{-1}(U) \cup f^{-1}(V)$. This shows that X is not αgs-connected space. This is contradiction. Therefore, Y is connected.

Recal that a function f: X→Y is said to be perfectly continuous [19] if $f^{-1}(U)$ is clopen in X for each open set U of Y.

Theorem 3.20: For two functions f: X→Y and g: Y→Z, let gof: X→Z is a composition function. Then, the following properties hold
(i) If f is almost contra αgs-continuous and g is an R-map, then gof is contra αgs-continuous.
(ii) If f is almost contra αgs-continuous and g is perfectly continuous,then gof is αgs-continuous and contra αgs-continuous.
(iii) If f is contra αgs-continuous and g is almost continuous, then gof is almost contra αgs-continuous.

Proof: (i) Let V be any regular open set in Z. Since g is an R-map, $g^{-1}(V)$ is regular open in Y. Since f is an almost contra αgs-continuous $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$ is αgs-closed sets in X. Therefore, gof is almost contra αgs-continuous.

(ii) Let V be any open set in Z. Since g is perfectly continuous, $g^{-1}(V)$ is clopen in Y. Since f is an almost contra αgs-continuous $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$ is αgs-open and αgs-closed set in X. Therefore, gof is αgs-continuous and contra αgs-continuous.

(iii) Let V be any regular open set in Z. Since g is almost continuous, $g^{-1}(V)$ is regular open in Y. Since f is contra αgs-continuous $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$ is αgs-closed sets in X. Therefore, g∘f is almost contra αgs-continuous.

Theorem 3.21: Let f: X→Y is almost contra αgs-continuous and g: Y→Z is αgs-continuous. If Y is $T_{\alpha gs}$-space, then gof: X→Z is almost contra αgs-continuous.

Proof: Let V be any regular open set in Z. Since g is αgs-continuous $g^{-1}(V)$ is αgs -open in Y and Y is $T_{\alpha gs}$-space implies $g^{-1}(V)$ open in Y. Since f is contra αgs-continuous $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$ is αgs-closed sets in X. Therefore, gof is almost contra αgs-continuous.

Theorem 3.22: If f: X→Y is surjective αgs-open (or αgs-closed) and g: Y→Z is a function such that gof: X→Z is almost contra αgs-continuous, then g is almost contra αgs-continuous.

Proof: Let V be any regular closed (resp. regular open) set in Z. Since gof is almost contra αgs-continuous, we have $(gof)^{-1}(V) = f^{-1}(g^{-1}(V)$ is αgs-open (resp. αgs-closed). Since f is surjective and αgs-open (or αgs-closed), we have $f(f^{-1}(g^{-1}(V))=g^{-1}(V)$ is αgs-open (or αgs-closed). Therefore g is almost contra αgs-continuous.

Definition 3.23: A f: X→Y is said to be αgs-open (resp. αgs-closed) if f(U) is αgs-open (resp. αgs-closed) in Y for each αgs-open (αgs-closed) set U of X.

Theorem 3.24: If f: X→Y is surjective αgs-open (αgs-closed) and g: Y→Z is a function such that gof: X→Z is almost contra αgs-continuous, then g is almost contra αgs-continuous.

Proof: Let V be a regular closed (resp. regular open) set in Z. Since gof is almost contra αgs-continuous, then $(gof)^{-1}(V) = f^{-1}(g^{-1}(V))$ is αgs-open (resp. αgs-closed). Since f is surjective and αgs-open (resp. αgs-closed), then $f(f^{-1}(g^{-1}(V)) = g^{-1}(V)$ is αgs-open (resp. αgs-closed). Therefore, g is almost contra αgs-continuous.

Definition 3.25: A topological space X is said to be αgs-ultra-connected if every two nonempty αgs-closed subsets of X intersect.

We recall that a topological space X is said to be hyperconnected [26] if every open set is dense.

Theorem 3.26: If X is αgs-ultra-connected and f: X→Y is an almost contra αgs-continuous surjection, then Y is hyperconnected.

Proof: Suppose Y is not hyperconnected. Then there exists an open set V such that V is not dense in Y. So there exist nonempty regular open subsets B_1=Int(Cl(V)) and B_2=Y-Cl(V) in Y. Since f is almost contra αgs-continuous, $f^{-1}(B_1)$ and $f^{-1}(B_2)$ are disjoint αgs-closed. This is contrary to the X is αgs-ultra-connected. Therefore, Y is hyperconnected.

Definition 3.27: A space X is said to be
(i) αgs-compact if every αgs-open cover of X has a finite sub cover.
(ii) Countably αgs-compact if every countable cover of X by αgs-open sets has a finite subcover.
(iii) αGS-closed if every αgs-closed cover of X has a finite subcover.
(iii) αgs-Lindelof if every αgs-open cover of X has a countable subcover.
(iv) αGS-Lindelof if every αgs-closed cover of X has a counta ble subcover.
(v) S-Lindelof [4] if every cover of X by regular closed sets has a countable subcover.
(vi) Countably αGS-closed if every countable cover of X by αgs-closed sets has a finite subcover.

Theorem 3.28: Let f: X→Y be an almost contra αgs-continuous surjection. Then, the following properties hold.
(i) If X is αGS-closed, then Y is nearly compact.
(ii) If X is Countably αGS-closed, then Y is nearly countabaly compact.
(iii) If X is αGS-Lindelof, then Y is Lindelof.

Proof: (i) Let $\{V_\alpha: \alpha\in I\}$ be any regular open cover of Y. Since f is almost contra αgs-continuous, then $\{f^{-1}(V_\alpha): \alpha\in I\}$ is αgs - closed cover of X. Since X is αGS-closed, there exists a finite subset I_o of I such that X= $\cup\{f^{-1}(V_\alpha): \alpha\in I_o\}$. Thus, we have Y= $\cup \{V_\alpha: \alpha\in I_o\}$ and Y is nearly compact.

(ii) Let $\{V_\alpha: \alpha\in I\}$ be any countable regular open cover of Y. Since f is almost contra αgs continuous, then $\{f^{-1}(V_\alpha): \alpha\in I\}$ is countable αgs-closed cover of X. Since X is countably αGS-closed, there exists a finite subset I_0 of I such that X= $\cup\{f^{-1}(V_\alpha): \alpha\in I_o\}$. Thus, we have Y= $\cup\{V_\alpha: \alpha\in I_o\}$ and Y is nearly countably compact.

(iii) Let $\{V_\alpha: \alpha\in I\}$ be any regular open cover of Y. Since f is almost contra αgs-continuous, then $\{f^{-1}(V_\alpha): \alpha\in I\}$ is αgs-closed cover of X. Since X is αGS-Lindelof, there exists a countable subset I_o of I such that X= $\cup\{f^{-1}(V_\alpha): \alpha\in I_o\}$. Thus, we have Y= $\cup\{V_\alpha: \alpha\in I_o\}$ and Y is nearly Lindelof.

Theorem 3.29: Let f: X→Y be an almost contra αgs-continuous surjection. Then, the following properties hold.
(i) If X is αgs-compact, then Y is S-closed.
(ii) If X is countably αgs-compact, then Y is countably S-closed.
(iii) If X is αgs-Lindelof, then Y is S-Lindelof.

Proof: (i) Let $\{V_\alpha: \alpha\in I\}$ be any regular closed cover of Y. Since f is almost contra αgs-continuous, then $\{f^{-1}(V_\alpha): \alpha\in I\}$ is αgs-open cover of X. Since X is αgs-compact, there exists a finite subset I_0 of I such that X= $\cup \{f^{-1}(V_\alpha): \alpha\in I_o\}$. Thus, we have Y= $\cup \{V_\alpha: \alpha\in I_o\}$ and Y is S-closed.

(ii) Let $\{V_\alpha: \alpha\in I\}$ be any countable regular closed cover of Y. Since f is almost contra αgs-continuous, then $\{f^{-1}(V_\alpha): \alpha\in I\}$ is countable αgs-open cover of X. Since X is countably αgs-compact, there exists a finite subset I_o of I such that X= $\cup \{f^{-1}(V_\alpha): \alpha\in I_o\}$. Thus, we have Y= $\cup \{V_\alpha: \alpha\in I_o\}$ and Y is countably S-closed.

(iii) Let $\{V_\alpha: \alpha\in I\}$ be any regular closed cover of Y. Since f is almost contra αgs-continuous, then $\{f^{-1}(V_\alpha): \alpha\in I\}$ is αgs-closed cover of X. Since X is αgs-Lindelof, there exists a countable subset I_0 of I such that X= $\cup \{f^{-1}(V_\alpha): \alpha\in I_o\}$. Thus, we have Y= $\cup \{V_\alpha: \alpha\in I_o\}$ and Y is S-Lindelof.

4. αGS-REGULAR GRAPHS

In this section, we introduce αGS-regular graphs and strongly contra αgs-closed graphs and investigate the relationship between the graphs and almost contra αgs-continuous functions.

Recall that for a function f: X→Y, the subset G_f = {(x, f(x))):

$x \in X\} \subset X \times Y$ said to be graph of f.

Definition 4.1: A graph G_f of a function f: X→Y is said to be αGS-regular (resp. strongly contra αgs-closed) if for each (x, y) ∈ (X x Y) \ G_f, there exist a αgs-closed (resp. αgs-open) set U in X containing x and V ∈ RO(Y) (resp. V∈ RC(Y)) containing y such that (U x V) ∩ G_f = ϕ .

Theorem 4.2: For a graph G_f of a function f: X →Y, the following properties are equivalent:

(i) G_f is αGS-regular (resp.strongly contra αgs-closed);

(ii) For each point (x, y) ∈ RO(Y)) (resp. V∈ RC(Y)) containing y such that f(U) ∩ V = ϕ .

Proof: This is direct consequences of Defininition 4.1 and the fact that for any subsets A ⊂ X and B ⊂ Y, (A x B) ∩ G_f = ϕ if and only if f(A) ∩ G_f = ϕ.

Theorem 4.3: If f: X→ Y is almost contra αgs-continuous and Y is T_2, then G_f is αGS-regular in X x Y.

Proof: Let (x, y) ∈ (X x Y) \ G_f. It is obvious that f(x) ≠ y. Since Y is T_2 there exist V, W ∈ RO(Y) such that f(x) ∈V, y ∈ W and V ∩ W = ϕ. Since f is almost contra αgs-continuous, $f^{-1}(V)$ is a αgs-closed set in X containing x. If we take U = $f^{-1}(V)$, then f(U) ⊂ V. Therefore, f(U) ∩ W = ϕ and G_f is αGS-regular.

Theorem 4.4: If f: X→ Y have αGS-regular graph G_f. If f is injective, then X is αgs-T_o.

Proof: Let x and y be any two distinct points of X. Then, we have (x, f(y)) ∈ (X x Y) \ G_f. Since G_f is αGS-regular, there exist a αgs-closed set U of X and V∈ RO(Y) such that (x, f(y)) ∈ (U x V) and f(U) ∩ W =ϕ by Theorem 4.2 and hence U ∩ $f^{-1}(V)$ = ϕ.Therefore, we have y ∉ U. Thus y∈ (X-U) and x ∉ (X-U). We obtain (X-U) ∈ αGSO(X). This implies that X is αgs-T_o.

Theorem 4.5: If f: X→ Y have αGS-regular graph G_f. If f is surjective, then Y is weakly Hausdorff.

Proof: Let y_1 and y_2 be any two distinct points of X. Since f is surjective, f(x) = y_1 for some x ∈ X and (x, y_2) ∈ (X x Y) \G_f. By Theorem 4.2, there exist a αgs-closed set U of X and F∈ RO(Y) such that (x, y_2) ∈ (U x F) and f(U) ∩ W = ϕ ; hence y_1 ∉ F. Then y_2 ∉ (Y-F) ∈ RC(Y) and y_1 ∈ (Y-F). This implies that Y is weakly Hausdorff.

5. Conclusion

The sets and functions in toplogical spaces and fuzzy topological spaces are extensively developed and used in many engineering problems, information systems, particle physics, computational topology and mathematical sciences.

By researching generalizations of closed sets, some new separation axioms have founded and are turned to be useful in the study of digital toplogy. Therefore, almost contra-αgs-continuous functions defined by αgs-closed sets will have many possibilities of applications in digital topology and computer graphics.

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